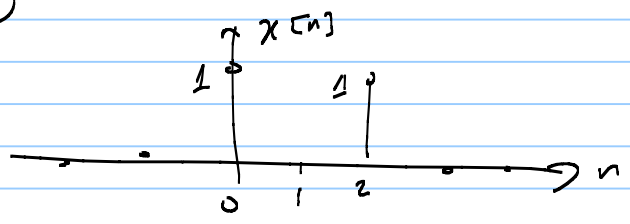
Note Title 4/3/2019

(1)




Compute the four-point DFT

"solution"

$N = 4$

$$X_k = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi kn}{N}}$$

$$= \sum_{n=0}^{3} x[n]\, e^{-j\frac{2\pi kn}{4}}$$

$$= x[0]\, e^{0} + x[1]\, e^{-j\frac{k\pi}{2}} + x[2]\, e^{-jk\pi} + x[3]\, e^{-j\frac{3k\pi}{2}}$$

$$= 1 + 0 + e^{-jk\pi} + 0$$

$e^{-jk\pi} = \cos(k\pi) - j\sin(k\pi)$

$= \cos(k\pi) + 0$

$X_k = 1 + \cos(k\pi)$

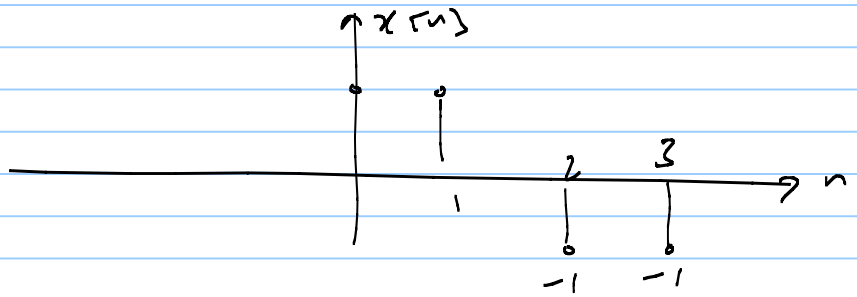
$X_0 = 1 + 1 = 2$

$X_1 = 1 + \cos(\pi) = 0$

$X_2 = 1 + \cos(2\pi) = 2$

$X_3 = 1 + \cos(3\pi) = 0$

---




Compute the four-point DFT

$$X_k = \sum_{n=0}^{3} x[n]\, e^{-j\frac{2\pi kn}{4}}$$

$$= 1\cdot e^{0} + 1\cdot e^{-j\frac{k\pi}{2}} - 1\cdot e^{-jk\pi} - 1\cdot e^{-j\frac{3k\pi}{2}}$$

$X_0 = 0$

$X_1 = 1 + e^{-j\frac{\pi}{2}} - e^{-j\pi} - e^{-j\frac{3\pi}{2}}$ $\longrightarrow \cos\left(\frac{\pi}{2}\right) - j\sin\left(\frac{\pi}{2}\right)$

$\longrightarrow \cos\left(\frac{3\pi}{2}\right) - j\sin\left(\frac{3\pi}{2}\right)$

$-j \quad -1 \quad j$

$= 1 - j + 1 - j = 2 - 2j$

$X_2 = 1 + e^{-j\pi} - e^{-j2\pi} - e^{-j3\pi}$

$-1 \quad 1 \quad -1$

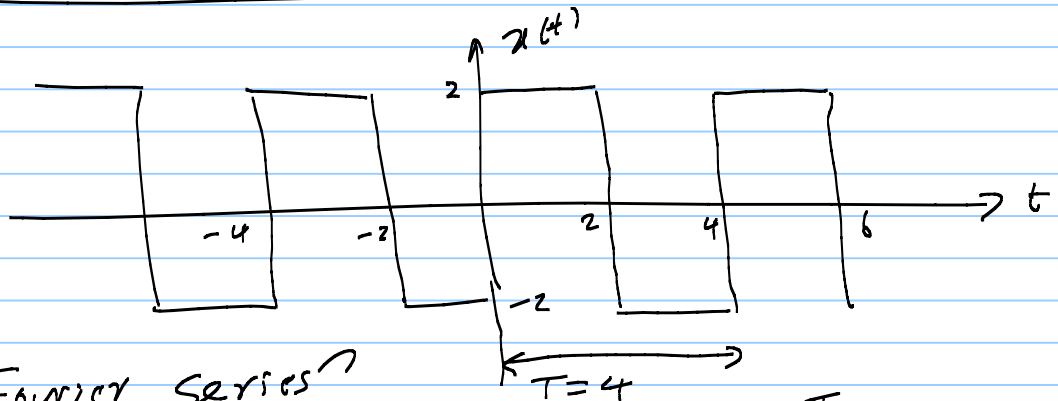
$X_2 = 1 - 1 - 1 + 1 = 0$

$X_3 = 1 + e^{-j\frac{3\pi}{2}} - e^{-j3\pi} - e^{-j\frac{9\pi}{2}}$

$j \quad -1 \quad -j$

$= 1 + j + 1 + j = 2 + j2$

---




Fourier series?

$\omega_0 = \frac{2\pi}{T} = \frac{\pi}{2}$

$\varphi_0 = 0$

$\varphi_k = 0$

$$b_k = \frac{4}{T}\int_0^{T/2} x(t)\sin(k\omega_0 t)\,dt$$

$$= \int_0^2 x(t)\sin\left(\frac{k\pi}{2}t\right)dt$$

$$= \int_0^2 2\sin\left(\frac{k\pi}{2}t\right)dt$$

$$= -\frac{4}{k\pi}\cos\left(\frac{k\pi}{2}t\right)\Big|_0^2 = -\frac{4}{k\pi}\left[\cos(k\pi) - \cos(0)\right]$$

$$= \frac{4}{k\pi}\left[1 - \cos(k\pi)\right]$$

$$b_k = \begin{cases} \frac{8}{k\pi}, & k = 1, 3, 5, 7, \cdots \\ 0, & k = 0, 2, 4, 6, \cdots \end{cases}$$

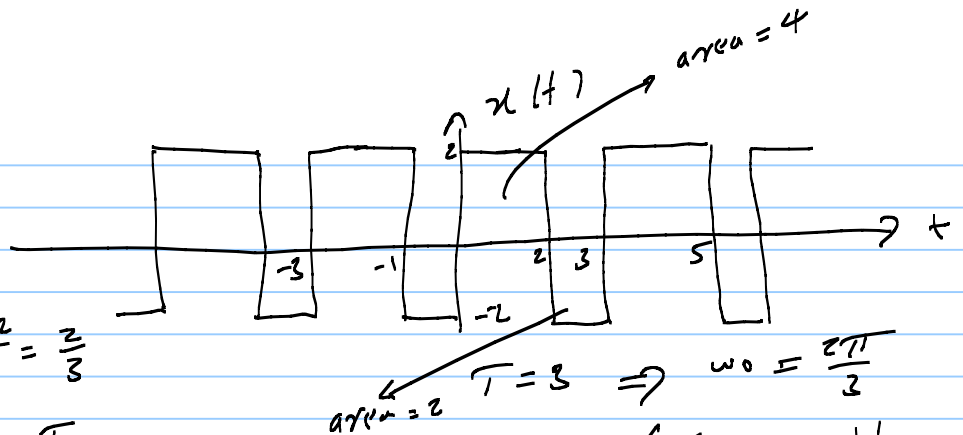
$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k\cos\left(k\frac{\pi}{2}t\right) + b_k\sin\left(k\frac{\pi}{2}t\right)$$

$$= \sum_{k=1}^{\infty}\frac{4}{k\pi}\left[1 - \cos(k\pi)\right]\sin\left(\frac{k\pi}{2}t\right) \qquad ; -\infty < t < \infty$$

OR $$x(t) = \sum_{\substack{k=1 \\ k\ \text{odd}}}^{\infty}\frac{8}{k\pi}\sin\left(\frac{k\pi}{2}t\right)$$

OR $$b_k = \frac{2}{T}\int_0^T x(t)\sin(k\omega_0 t)\,dt$$

$$= \frac{2}{4}\left[\int_0^2 2\sin\left(\frac{k\pi}{2}t\right)dt - \int_2^4 2\sin\left(\frac{k\pi}{2}t\right)dt\right]$$




$a_0 = \frac{4-2}{3} = \frac{2}{3}$ $\quad T = 3 \Rightarrow \omega_0 = \frac{2\pi}{3}$

~~$a_k = 0$~~ ← odd

$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt$$

$$= \frac{1}{3}\left[\int_0^2 2\,dt - \int_2^3 2\,dt\right] = \frac{1}{3}\left[2t\Big|_0^2 - 2t\Big|_2^3\right]$$

$$= \frac{1}{3}\left[(4-0) - (6-4)\right] = \frac{1}{3}\left[4-2\right] = \frac{2}{3}$$

$$a_k = \frac{2}{3}\left[\int_0^2 2\cos\left(\frac{2k\pi}{3}t\right)dt - \int_2^3 2\cos\left(\frac{2k\pi}{3}t\right)dt\right]$$

$=$

$$b_k = \frac{2}{3}\left[\int_0^2 2\sin\left(\frac{2k\pi}{3}t\right)dt - \int_2^3 2\sin\left(\frac{2\pi k}{3}t\right)dt\right)$$

$=$

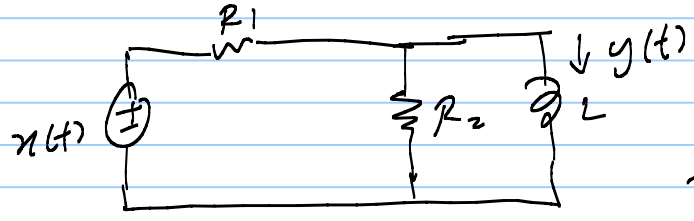
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Even $\quad b_k = 0$

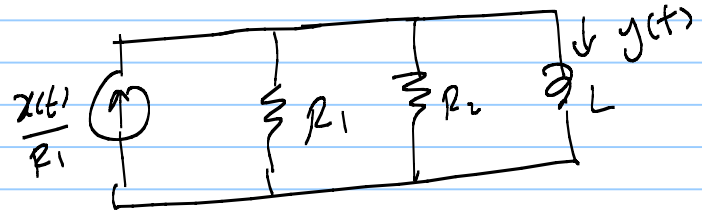
Odd $\quad a_0 = 0$ ; $a_k = 0$

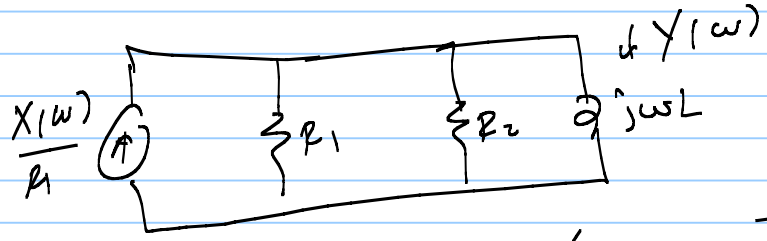
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$x(t) = 10\cos(1000t + 20°)$




find $y(t)$




$$Y(w) = \frac{X(w)}{R_1}\left(\frac{\frac{1}{jwL}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{jwL}}\right) \times \frac{R_1R_2\,jwL}{R_1R_2\,jwL}$$

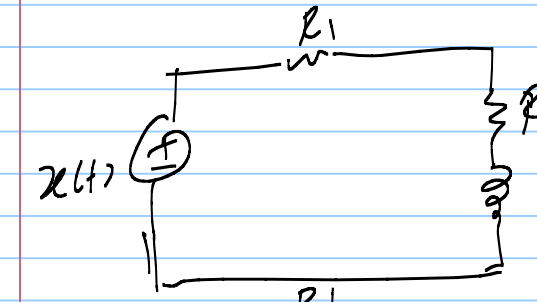
$$= \frac{X(w)}{R_1}\left(\frac{R_1R_2}{jwR_2L+jwR_1L+R_1R_2}\right)$$

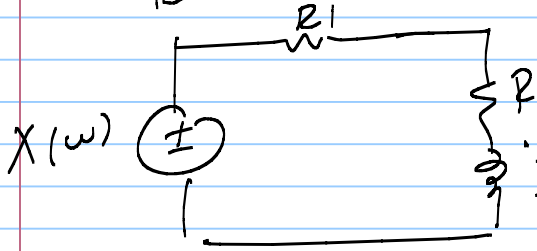
$$= X(w)\,\frac{R_2}{R_1R_2+jwL(R_1+R_2)}$$

$$H(w) = \frac{Y(w)}{X(w)} = \frac{R_2}{R_1R_2+jwL(R_1+R_2)}$$

$$y(t) = 10\left|\frac{R_2}{R_1R_2+j1000L(R_1+R_2)}\right|\cos\left(1000t+20^\circ-\tan^{-1}\left(\frac{1000L(R_1+R_2)}{R_1R_2}\right)\right)$$

---



$$H(\omega) = \frac{R_2 + j\omega L}{R_1 + R_2 + j\omega L}$$

$$\angle H(\omega) = \tan^{-1}\left(\frac{\omega L}{R_2}\right) - \tan^{-1}\left(\frac{\omega L}{R_1 + R_2}\right)$$

$$|H(\omega)| = \frac{\sqrt{R_2^2 + \omega^2 L^2}}{\sqrt{(R_1+R_2)^2 + \omega^2 L^2}}$$

$$x(t) = 10 + 20\cos(1000t + 10^\circ) + 15\sin(30t + 90^\circ)$$

$$= 10\left(\frac{R_2}{R_1+R_2}\right) + 20\left|\frac{\sqrt{R_2^2 + 10^6 L^2}}{\sqrt{(R_1+R_2)^2 + 10^6 L^2}}\right|\cos(1000t + 10^\circ + \angle H(1000))$$

$$+ 15\left(|H(30)|\right)\sin(30t + 90^\circ + \angle H(30))$$

---

$$X(\omega) = \frac{1}{j}\left[2\,\text{sinc}\left(\frac{2\omega}{\pi} - \frac{1}{4}\right) - 2\,\text{sinc}\left(\frac{2\omega}{\pi} + \frac{1}{4}\right)\right]$$

$$= \frac{-1}{j}\left[2\,\text{sinc}\left(\frac{2\omega}{\pi} + \frac{1}{4}\right) - 2\,\text{sinc}\left(\frac{2\omega}{\pi} - \frac{1}{4}\right)\right]$$

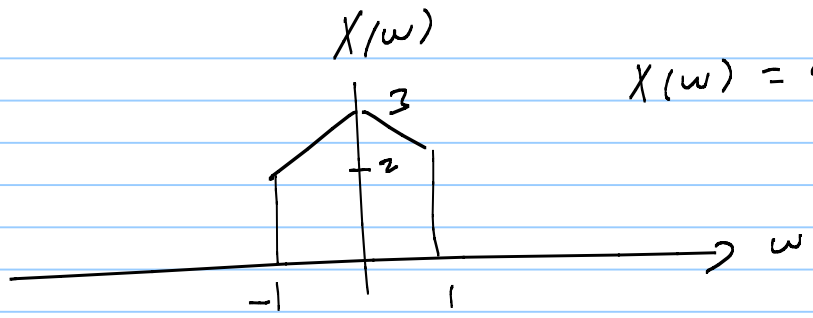
$$= \frac{-1}{j}\left[2\,\text{sinc}\left(\frac{2\omega + \frac{\pi}{4}}{\pi}\right) - 2\,\text{sinc}\left(\frac{2\omega - \frac{\pi}{4}}{\pi}\right)\right]$$

$$= j\left[2\,\text{sinc}\left(\frac{2(\omega + \frac{\pi}{8})}{\pi}\right) - 2\,\text{sinc}\left(\frac{2(\omega - \frac{\pi}{8})}{\pi}\right)\right]$$

$$P_4(t) \leftrightarrow 4\,\text{sinc}\left(\frac{4\omega}{2\pi}\right) = 4\,\text{sinc}\left(\frac{2\omega}{\pi}\right)$$

$$P_4(t)\sin\left(\frac{\pi}{8}t\right) \leftrightarrow \frac{j}{2}\left[4\,\text{sinc}\left(\frac{2(\omega + \frac{\pi}{8})}{\pi}\right) - 4\,\text{sinc}\left(\frac{2(\omega - \frac{\pi}{8})}{\pi}\right)\right]$$

$$x(t) = P_4(t)\sin\left(\frac{\pi}{8}t\right)$$




$$X(\omega) = 2P_2(\omega) + \left(1 - \frac{2|\omega|}{2}\right) P_2(\omega)$$

find $x(t)$